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NRL Report 7940

Greenhill's Formula and the Mechanics of Cable Hockling

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November 7, 1975

NAVAL RESEARCH LABORATORY
Washington, D.C.

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JUSTIFICATION

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

14

1. REPORT NUMBER: NRL ~~Report~~ 7940

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

9

4. TITLE (and Subtitle): GREENHILL'S FORMULA AND THE MECHANICS OF CABLE HOCKLING.

6

5. TYPE OF REPORT & PERIOD COVERED: Final report ~~on one phase of a continuing NRL~~ problem.

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s): Felix Rosenthal

10

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Research Laboratory, Washington, D.C. 20375

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NRL Problem F02-24, Project RR 023-03-45-5807

11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Arlington, Va. 22217

11

12. REPORT DATE: Nov[redacted]75

13. NUMBER OF PAGES: 29

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

12

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

RR023-03, NRL-F02-24

18. SUPPLEMENTARY NOTES

RR023-03-45

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Ocean engineering
Cables
Elasticity theory
Rods
Hockling
Kinking
Wire rope

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In the application of marine cables as tension members, serious structural failures have occurred as a result of hockling (forming of a loop under torsion combined with insufficient tension). This report endeavors to establish a criterion on end torques and forces for preventing hockling. It is shown that for rods in tension the torque corresponding to Greenhill's 100-year-old formula defining the onset of bending is also the maximum torque which can be sustained at that tension. Thus, a straight rod or cable loaded by tension and the Greenhill torque is unstable, and Greenhill's formula modified by an appropriate safety factor provides the desired no-hockle criterion.
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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

251 950

20. Abstract (Continued)

For long straight rods or cables under tension T and twisting moment M, this criterion for stability is $M^2 < 4TEI$, where EI is the bending stiffness under load. The approach used was to numerically solve in nondimensional form the two-point boundary-value problem of the rod under axial end torques and forces. The report includes curves showing torques and forces for the possible range of deflection curves.

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GREENHILL'S FORMULA AND THE MECHANICS OF CABLE HOCKLING

INTRODUCTION

In the application of marine cables as tension members, such as in lifting objects from the ocean floor, serious structural failures have occurred as a result of two phenomena called hockling and bird caging. Both problems stem from the torsional moments which develop in the cable, usually as a result of its tendency to unwind under load. Hockling occurs when, for a given torque reaction at the ends of the cable, the tension becomes insufficient to keep the cable taut, resulting in the formation of a loop or hockle. When increased tension is subsequently reapplied to a hockled cable, the loop tends to tighten, causing the cable to fail. Bird caging, on the other hand, occurs when individual wires or strands unravel under excessive torsional loads that are reverse to the direction in which the wire strands are wound.

The hockling of cables would seem to be closely related to the well-known problem of the elastica (thin elastic rod under combined tension and moments applied at its ends). This problem has received much attention in the literature. Kirchhoff [1,2] recognized that the deflection curve of the elastica is governed by the same set of differential equations as the motion of a heavy spinning top. Greenhill [2-4] gave a buckling formula for the rod subjected to tension (or thrust, if negative) and twisting couples, based on the assumption of infinitesimal bending deformations. Southwell [5], in a discussion of the elastica under end forces with zero moments, showed that a column under thrust at and above the Euler buckling load remains stable. More recently, considerable effort has gone into studies of the dynamic characteristics of thin three-dimensional beams. An excellent review and bibliography of this subject through 1972 may be found in Ref. 6. The problem of rods of variable cross section has been comprehensively addressed by Green, Naghdi, and Wenner [7].

In spite of all this attention to the problem of the elastica, some puzzling questions have remained with regard to the hockling of cables: What are the stability characteristics of small-deflection solutions corresponding to Greenhill's formula with respect to increasing torque or decreasing tension? What in fact is the largest torque for given tension to which a cable may safely be subjected? And, conversely, how far may the tension safely be lowered for given end torque? When this report's author first became interested in the hockling problem, he did a rudimentary experiment, trying to put bending deflections into a straight rod under twist and modest tension. The rod was twisted until numerous helical slip lines developed, indicating substantial shear yielding. Yet no significant bending deflections were observed. But cables in tension do hockle. Must they be described in a way essentially different from rods, or can the observed difference in behavior be explained entirely in terms of the much lower bending stiffness pertaining to cables?

Manuscript submitted October 3, 1975.

The purpose of the analysis described in this report was to obtain some insight into these questions and if possible to establish a loading criterion for the avoidance of cable hockles. The approach was to obtain computer solutions to the differential equations of the elastica in nondimensional form for the full range of values of axial end forces and moments.

STATEMENT OF THE PROBLEM

The problem is to calculate the deflection curve and strain energy of a cable or rod that is prismatic and straight when unloaded and is "hinged" at each end so as to permit only an axial moment **M** and an axial force **T**. *Axial* means along the straight line joining the two end points. Although this presents a two-point boundary-value problem, it can be solved in reverse as an initial-value problem by assuming a starting angle γ between the deflection curve and the axis and then solving the differential equations step by step until a point possessing appropriate symmetry properties is reached. This point (or alternatively the end point) must be established from the local properties of the deflection curve, since the length is initially unknown and must be calculated. When the problem is solved in nondimensional form, a two-parameter set of solutions is sufficient to cover the full range of possible loading values.

DERIVATION OF EQUATIONS

A length ℓ of rod or cable that is straight and prismatic when unloaded and possesses a bending stiffness EI and torsional rigidity GJ is assumed to lie along a space curve and is referred to a fixed coordinate system $OXYZ$ with unit vectors **I**, **J**, and **K** as shown in Fig. 1. The $+X$ axis is vertical upward. Arc length along the cable is denoted by s and increases as shown in the figure. The cable is assumed to be "hinged" at O through something resembling a universal joint such that the reaction there consists solely of a vertical force **T** and a vertical couple **M**, both directed downward if positive. T and M denote the magnitudes of **T** and **M**(0) respectively. At the free end P of the cable, where $\mathbf{x}(s) = [x, y, z]$, the required equilibrating forces are a constant vertical upward force **T** and a couple **M**(s), where

$$\mathbf{T}(s) \equiv \mathbf{T}(0) = \mathbf{T} = T\mathbf{I} \tag{1a}$$

and

$$\mathbf{M}(s) = \mathbf{M}(0) + \mathbf{T} \times \mathbf{x}(s) = M\mathbf{I} - Tz\mathbf{J} + Ty\mathbf{K}. \tag{1b}$$

At the free end P of the cable segment OP, Fig. 1 shows the unit vectors of two other coordinate systems: $\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}$ and $\mathbf{t}, \mathbf{n}, \mathbf{b}$. Here $\boldsymbol{\zeta} = \mathbf{t}$ is the unit tangent vector to the space curve defined by the cable's center line. The vectors $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are perpendicular to the tangent and thus lie in the plane of the cross section of the cable. They are taken as principal axes of the cross section and are considered to be inscribed in the cross section (rotate with the cable).

On the other hand, $\mathbf{n}$ and $\mathbf{b}$ are respectively the principal normal and the binormal of the center line.

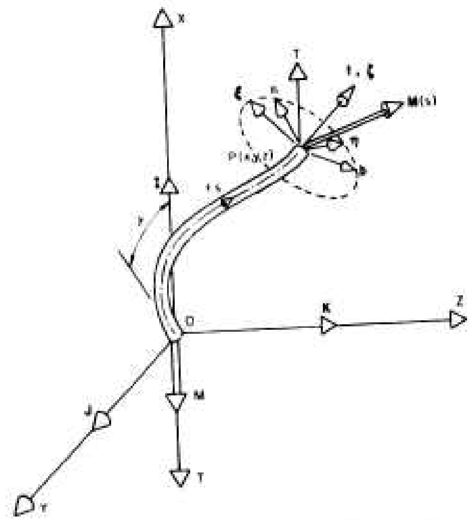

Fig. 1 — Coordinate systems and free-body diagram of cable section OP

It is usual [8] to introduce an "angular velocity" vector $\boldsymbol{\Omega}$ which denotes the vector rate of turn of the system ξ, η, ζ with respect to arc length s.

Since $\mathbf{t}$ is a unit vector, its arc-length derivative is thus given by

$$d\mathbf{t}/ds \equiv \mathbf{t}' = \boldsymbol{\Omega} \times \mathbf{t}.$$

Cross multiplication by $\mathbf{t}$ yields

$$\boldsymbol{\Omega} = K\mathbf{b} + \Omega_\zeta \mathbf{t},$$

where $\mathbf{n}$ and $\mathbf{b}$ are defined by

$$\mathbf{t}' = \boldsymbol{\Omega} \times \mathbf{t} = K\mathbf{n} \tag{2}$$

and

$$\mathbf{b} = \mathbf{t} \times \mathbf{n}$$

and $\Omega_b = K$ is the principal curvature of the center line. In terms of the local components of the moment vector $\mathbf{M}$ the components of Ω are

$$\Omega_t = M_t/GJ, \tag{3a}$$

$$\Omega_n = M_n = 0, \tag{3b}$$

and

$$\Omega_b = M_b/EI = K. \tag{3c}$$

Differentiation of (2) further yields the derivative of the normal vector,

$$\mathbf{n}' = \mathbf{b}' \times \mathbf{t} - K\mathbf{t}. \tag{4}$$

Taking components of (4) in the $\mathbf{t}$ and $\mathbf{b}$ directions respectively gives

$$\mathbf{n}' \cdot \mathbf{t} = -K$$

and

$$\mathbf{n}' \cdot \mathbf{b} = -\mathbf{b}' \cdot \mathbf{n}. \tag{5}$$

The quantities $\mathbf{n}' \cdot \mathbf{n}$ and $\mathbf{b}' \cdot \mathbf{b}$ on the other hand are zero, since they represent the component of the derivative of a unit vector upon itself. The remaining required derivative component is $\mathbf{b}' \cdot \mathbf{t}$, which is zero as shown by the following: Since $\mathbf{t} \cdot \mathbf{b} \equiv 0$, it follows that $\mathbf{t}' \cdot \mathbf{b} + \mathbf{t} \cdot \mathbf{b}' = 0$. But since $\mathbf{t}'$ is perpendicular to $\mathbf{b}$, then $\mathbf{t}' \cdot \mathbf{b}$ and hence also $\mathbf{t} \cdot \mathbf{b}'$ must vanish. The preceding calculation of the components of the $\mathbf{t}'$, $\mathbf{n}'$, $\mathbf{b}'$ vectors along the $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ coordinates may be summarized as the matrix equation representing the Frenet-Serret differential equations for a space curve:

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}' \\ \mathbf{b}' \end{bmatrix} = \begin{bmatrix} 0 & K & 0 \\ -K & 0 & \varphi \\ 0 & -\varphi & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix}. \tag{6}$$

The quantity φ, substituted for $\mathbf{n}' \cdot \mathbf{b}$ in (5), is thus seen to be the geometric torsion, or *tortuosity* as Love [2] calls it, of the space curve traced by the center line of the cable or rod. Differentiating equation (1b), noting that $\mathbf{x}' \equiv \mathbf{t}$ and using equations (3), gives along the $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ components

$$\mathbf{M}' \cdot \mathbf{t} = M_t' \equiv 0, \tag{7a}$$

$$K(\varphi EI - M_t) + \mathbf{T} \cdot \mathbf{b} = 0, \tag{7b}$$

and

$$EIK' = -\mathbf{T} \cdot \mathbf{n}. \tag{7c}$$

Equation (7a) states that the internal twisting moment, and hence the twist angle per unit length, is constant along the arc length. $M_t \equiv M_t(0)$ is one of the three first integrals available to this problem, analogously with the problem of the heavy top. Equation (7b) is needed along with equations (3) and (6) to complete the system of equations.

The following non-dimensional quantities are introduced:

$$S = Ts/M, \;\; X = Tx/M, \;\; Y = Ty/M, \;\; Z = Tz/M, \;\; L = T\ell/M;$$

$$F = M^2/TEI, \;\; H = KEI/M, \;\; \Phi = \varphi EI/M,$$

$$(^*) = d/dS = (M/T)\,(\,'\,), \;\; M_T = M_t/M \equiv \alpha_{11}\,(0).$$

Further the direction cosines, or components α_{ij} of the $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ vectors with respect to the fixed system $\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$, as well as a set of Euler angles A, B, C to remove unncessary redundacies in the direction cosines, are defined:

$$\alpha_{11} = \cos\,(\mathbf{t}, \mathbf{I}),$$

$$\alpha_{12} = \cos\,(\mathbf{t}, \mathbf{J}),$$

etc., and

$$[\alpha_{ij}] = \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} cBcC & cBsC & -sB \\ sAsBcC - cAsC & sAsBsC + cAcC & sAcB \\ cAsBcC + sAsC & cAsBsC - sAcC & cAcB \end{bmatrix}, \tag{8}$$

where cB means $\cos B$, sC means $\sin C$, etc.

In these terms the complete system of differential equations defining the space curve and other quantities pertaining to the rod or cable consists of equation (8) for the α_{ij} in terms of the Euler angles together with the following:

$$H = \alpha_{31} - Z\alpha_{32} + Y\alpha_{33}, \tag{9a}$$

$$\Phi = M_T - \alpha_{31}/FH, \quad \text{if } H \neq 0, \tag{9b}$$

$$\Phi = M_T/2, \quad \text{if } H = 0 \text{ (by L'Hospital's rule)}, \tag{9c}$$

$$A^* = F(HcAsB/cB + \Phi), \tag{9d}$$

$$B^* = -FHsA, \tag{9e}$$

$$C^* = FHcA/cB, \tag{9f}$$

$$X^* = \alpha_{11}, \tag{9g}$$

$$Y^* = \alpha_{12}, \tag{9h}$$

$$Z^* = \alpha_{13}. \tag{9i}$$

INITIAL CONDITIONS

Initial conditions are chosen with $X_0 = Y_0 = Z_0 = 0$ and so that the tangent vector initially makes an angle γ $(0 \leqslant \gamma \leqslant 180^\circ)$ with the X axis. An appropriate set of initial values of the Euler angles is

$$A_0 = \pi/2,$$

$$B_0 = 0,$$

$$C_0 = \gamma,$$

thus making the curve begin in the XY plane.

Equations (9) become singular when $cB = 0$. This happens infrequently, and is entirely a difficulty of the particular Euler-angle representation. It is avoided either by choosing a different initial azimuth (say $A_0 = 0$, $B_0 = \gamma$, $C_0 = 0$) or by departing from the Euler-angle formulation whenever cB is small and instead using derivatives of a set of three independent direction cosines.

FINDING MIDHOCKLE

To determine when midhockle, the point of symmetry halfway between the "hinged" ends, is reached, we seek an arc length s_m at which $K(s_m + s) = K(s_m - s)$ and $\varphi(s_m + s) = \varphi(s_m - s)$ for any s. This happens when all the odd derivatives $K^{(2N+1)}(s)$ and $\varphi^{(2N+1)}(s)$ are equal to zero at $s = s_m$. It is easily shown that this happens whenever $K'(s) = 0$ or, by equation (7c), when $\alpha_{21} = 0$.

An outline of the proof comes in two parts: The first part to be proved is that if $K^{(2n+1)} = \varphi^{(2n+1)} = 0$ for $n = 0, 1, ..., N-1$ and if $K^{(2N+1)} = 0$, then also $\varphi^{(2N+1)} = 0$. This follows from successive double differentiations of (7b) and use of (6) and (7c). Each such doubly differentiated equation consists of terms which are products of derivatives of order zero up to the order of differentiation. Since the sum of orders for each term is odd, each term must contain at least one factor which is an odd-order derivative of either K or φ. But by hypothesis all these factors except the highest derivative of φ are zero. Hence also the highest derivative must vanish.

The second part to be proved is that if $K^{(2n+1)} = \varphi^{(2n+1)} = 0$ for $n = 0, ..., N-1$, then also $K^{(2N+1)} = 0$. Since by (7c) differentiated $2n$ times

$$EIK^{(2n+1)} = -\mathbf{T} \cdot \mathbf{n}^{(2n)},$$

it suffices to show that $-\mathbf{T} \cdot \mathbf{n}^{(2N)} = 0$, or that $\mathbf{n}^{(2N)}$ has a zero $\mathbf{l}$ component. Let the direction-cosine matrix of (8) be denoted by τ and the curvature-torsion matrix of (6) by χ:

$$\tau = \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix}$$

and

$$\chi = \begin{bmatrix} 0 & K & 0 \\ -K & 0 & \varphi \\ 0 & -\varphi & 0 \end{bmatrix}$$

Then (6) may be written as

$$\tau' = \chi\tau. \tag{10}$$

To be shown therefore is that under the assumed circumstances the $\{2,1\}$ component of matrix $\tau^{(2N)}$ is zero. Successive differentiation and substitution of (10) shows that the higher derivatives of τ are expressible in the form

$$\tau^{(n)} = \left[f_n(\chi^{(n-1)}, \ldots, \chi', \chi) \right] \tau,$$

where each term of f_n is a product of from one to n of the n matrices χ through $\chi^{(n-1)}$. From the formal process of differentiation it follows that these terms are homogeneous of degree n in each term's sum of orders plus number of factors. As an example a term $\chi\chi'''\chi'\chi\chi$ has a total order $0 + 3 + 1 + 0 + 0 = 4$ and has five factors and therefore could belong only to f_9. Thus for $n = 2N$ the highest derivative term is χ^{2N-1} multiplied by no other factors. Because of the homogeneity of orders plus factors of degree $2N$, every term of f_{2N} other than this highest derivative term must also possess one of two properties: Either it is the product of an even number of factors or it must contain at least one lower odd-order derivative. In either case, the matrix term can be shown to be zero at all "odd" locations including $\{2,1\}$; hence the same follows for their sum $\tau^{(2N)}$.

From the preceding two parts of the proof it follows that a point at which $K' = 0$ (or $\alpha_{21} = 0$) is always a point of symmetry.

BRIEF DESCRIPTION OF THE FORTRAN LISTINGS

The Fortran routine to accomplish the required calculations consists of the main program Hockle and subroutines Dcalc, Sym, and Step. The listings are given in Appendix A.

The main program accepts sets of values for F, γ, increment of nondimensional arc length ΔS, and print interval (to permit printing less than every calculation step). It also calculates initial conditions and determines when midhockle is passed (α_{21} changes sign), at which time it interpolates for S_m. *Endhockle* is prescribed to be at $S = L = 2S_m$, and the option exists to keep calculating that far. The program Hockle contains necessary print and punch instructions and calls subroutines as needed. An important output is sets of values of nondimensional moment $u = M\ell/EI = FL$ and force $v = T\ell^2/EI = FL^2$.

The subroutine Dcalc calculates the distance of closest approach D_{min} between pairs of symmetric points on the rod. This is done because if that distance became zero in a real cable, then the mutual lateral forces at the point of intersection would no longer permit representation by the present mathematical model, which would allow the cable to pass right through itself. The calculation of D_{min} is performed using points only between zero and midhockle and does not require stepping past midhockle.

The subroutine Sym does require calculation all the way to endhockle and calculates the degree of symmetry about midhockle of selected quantities such as Φ and α_{21}. This provides a check on the accuracy of calculation, since in theory the symmetry should be perfect.

Finally, the subroutine Step performs the basic stepping procedure on the differential equations and calculates the required quantities: direction cosines, curvature H (both directly and from a first integral for comparison), Φ, etc. When informed by Hockle that cB is too small to use the Euler angles, Step uses alternate equations for stepping. The stepping interval is the inputted ΔS except either side of midhockle, where steps and print points are chosen in such a way as to preserve symmetry of the calculations and print intervals.

DISCUSSION OF RESULTS

Relation Between End Forces, End Moments, and Cable Configuration

Once the program has calculated the nondimensional length L of a symmetric space curve for given values of load parameter F and initial angle γ, then the values of the nondimensional moment and force

$$u = M\ell/EI = FL$$

and

$$v = T\ell^2/EI = FL^2$$

are readily calculated. The results are shown in Fig. 2 for a number of values of γ, with a few F = constant curves also indicated. The $\gamma = 0°$ and $\gamma = 180°$ curves constitute the two halves of a parabola corresponding to Greenhill's formula

$$v = \left| (u/2)^2 - \pi^2 \right|, \tag{11}$$

referred to in the Introduction. The reason for the absolute-value signs and thus the parabola with the lower portion reflected in the u axis is that there is the following duality of solutions: If a (F, γ) input yields a (u, v) solution, then a $(-F, \pi-\gamma)$ input yields the identical $(u, -v)$ solution. However the concept of positive F and T representing a tension and negative F and T representing a compression makes sense only for the Greenhill case of $\gamma = 0$ (or, for reversed signs, $180°$). For example, the elastica, under end force only, can go with a continuously increasing force from Euler buckling in compression, at $(0, \pi^2)$ in the figure, to a looped rod in tension, with no clear break in between. To exhibit this behavior correctly, all forces as well as moments are therefore shown in the first quadrant, with the result that half of the Greenhill parabola is reflected in the u axis. Also, because of these same symmetries, all γ = constant curves are perpendicular to the v axis at $u = 0$. In addition the $\gamma = 90°$ curve is perpendicular to the u axis at $v = 0$.

An important set of check points is provided on the v axis. Clearly this corresponds to the elastica with zero moment. As shown by Southwell [5], the relation on $u = 0$ between v and γ is given by

$$v = 4\mathfrak{K}^2(k),$$

where

$$k = \sin[(\pi-\gamma)/2] \text{ and}$$

where $\mathfrak{K}(k)$ is the complete elliptic integral of the first kind. This formula checks the computer solution for small values of F and hence of u.

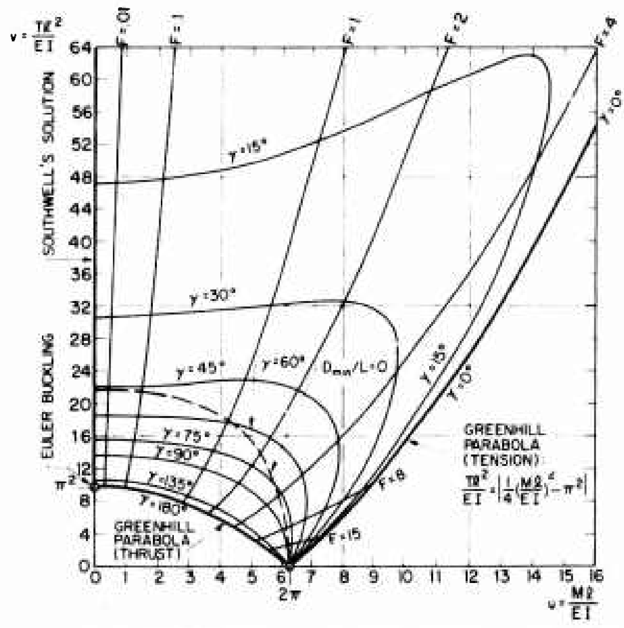

Fig. 2 — Nondimensional end moments and forces on an elastica for various end angles γ and load parameters $F = M^2/TEI$ or u^2/v

In Fig. 2 all nontrivial solutions to this problem as expressed in the uv quarter plane fall between the two segments of the Greenhill parabola. (Trivial solutions exist throughout the uv quarter plane.) All combinations of moment and force inside the 180° segment of the parabola correspond to the trivial solution of twist only, and zero bending. Similarly moments outside the 0° segment are unattainable, since only an unstable trivial solution exists there.

The dashed curve $D_{\min}/L = 0$ along with the portion of the v axis above 21.55 is the locus in uv space along which the calculated deflection curve of the loaded cable possesses a self-intersection. This locus provides a barrier to the configuration of a real cable. For example, if a cable loaded by a constant thrust $v = 20$ is subjected to an increasing moment u, a self-intersection will occur when u equals approximately 3, and any configuration corresponding to larger moments will not be correctly predicted by the present theory. With reference again to Southwell's solution, the $D_{\min}/L = 0$ curve meets the v axis when the ratio of the complete elliptic integral of the second kind $\mathcal{E}(k)$ to the one of the first kind equals 1/2:

$$\mathcal{E}(k)/\mathcal{K}(k) = 1/2.$$

This in turn occurs when $v = 21.55$ and $\gamma = 49.29°$ as shown in Fig. 2.

If a rod is subjected to a constant compressive force below the Euler load while the twisting moment is increased starting at zero, the rod remains straight until the moment reaches the lower Greenhill value at $\gamma = 180°$. This point is stable however in that the moment can be increased beyond this value at least until the dashed curve is reached.

If a rod or cable begins in tension, it remains straight until the upper Greenhill value is reached at $\gamma = 0°$, but at this point no further increase in torque is possible and any further twisting deformation must be accompanied by a decreased reaction torque. Thus for a rod or cable in combined tension and twist, Greenhill's formula does not represent merely the point of first departure from a trivial twist-only solution. It additionally represents the maximum torque which can be applied at a given tension, or the minimum tension necessary to support a given torque, and represents a point of instability if any attempts were made to increase the torque relative to the force. This instability must be particularly violent in the case of cables, for the following reason: For a rod or cable in combined tension and sub-Greenhill torsion, all strain energy is stored in twist, a relatively stiff mode. As the Greenhill curve is approached and bending becomes possible, a substantial part of this twisting strain energy must be transferred into bending. But a cable is distinguished by its very low bending stiffness, so that the conversion of a given amount of strain energy would require much larger deflections than would be the case for a rod.

The previous paragraphs also illustrate the double-valuedness of the graph of Fig. 2 in the area between the two branches of the Greenhill curve. Consider a cable loaded by a nondimensional moment $u = 7.2$ and a nondimensional force $v = 52.4$, at the point in Fig. 2 at which $\gamma = 15°$ and $F = 1$. One possible configuration of the cable under this loading is the straight cable in tension, with γ actually equal to $0°$ and not $15°$. This configuration can result if the tension on an initially unloaded cable is raised to 52.4 and a subsequently applied moment is raised to 7.2. Since this is below the Greenhill torque in tension, the cable remains straight. The other possible configuration under the same loading is a hockled cable with γ really equal to $15°$. This may be produced by "compressing" an initially straight cable past Euler buckling, forming a loop which is tightened as the force is increased to 52.4 (now a "tension" because the end points have passed each other), with a subsequent increase in moment to 7.2, applied in the direction in which the loop is able to open partially.

The quarter plane $u \geqslant 0$, $v \geqslant 0$ is thus divided into three distinct regions: Inside the $180°$ branch of the Greenhill curve, only the trivial solution exists, and it is stable. In the second region, between the $180°$ and $0°$ branches, the configuration is double valued for any given loading. Here, if loading begins from a straight rod in tension, then the trivial solution is stable, but if the loading begins from compression, then the trivial solution is unstable and the nontrivial configuration results. In the third region, outside the $0°$ branch of the Greenhill curve, only the unstable trivial solution exists, so that such loadings cannot be sustained.

It is instructive to examine from an energy point of view the possible rod or cable configurations at the load point in Fig. 2 ($u = 2\pi$, $v = 0$) where all the γ = constant curves intersect. The loading corresponding to this point supports equilibrium configurations for all values of γ from $0°$ to $180°$. The deflection curve corresponding to these configurations would in general be a complete turn of a helix of pitch $\pi/2 - \gamma$. It is a helix because

the force is zero and the internal moment therefore is constant; it contains a full revolution because the end moments act along the direction of the line joining the end points. For $\gamma = 0°$ or $180°$, the helix is reduced to a straight line in pure twist under end moment M; for $\gamma = 90°$, it is a full circular hoop in pure bending under end moments M perpendicular to the plane of the circle. Of all possible equilibrium configurations, the stable one is that one for which the applied end moments are at the lowest possible level of potential energy, i.e., the one for which the strain energy is maximum.

For this case of pure moment loading, the total strain energy is

$$U = M_b^2 \ell/2EI + M_t^2 \ell/2GJ,$$

where GJ is the torsional stiffness. In terms of the applied moment M and the angle γ, $M_b = M \sin \gamma$ and $M_t = M \cos \gamma$, so that

$$U = M^2\ell/2 \; (\sin^2 \gamma/EI + \cos^2 \gamma/GJ).$$

The extreme values of U occur when $dU/d\gamma = 0$, or when

$$\sin 2\gamma \; (1/EI \cdot 1/GJ) = 0,$$

or $\gamma = 0°$, $90°$, and $180°$. Which extreme corresponds to the maximum strain energy then depends on the stiffnesses, as may be seen by putting the three values of γ back into U:

$$U \; (\gamma = 0°, 180°) = M^2\ell/2GJ, \text{ for pure twist,}$$

and

$$U \; (\gamma = 90°) = M^2\ell/2EI, \text{ for pure bending.}$$

Thus for the rod or cable loaded by end moments $M\ell/EI = 2\pi$, the stable configuration is the straight twisted one if $GJ < EI$ and is the circular hoop if $GJ > EI$. For typical solid rods, $GJ \approx 0.7EI$, while for cables $GJ >> EI$. Thus a hinged cable loaded with $v = 0$ and u increasing from zero will remain straight until $u = 2\pi$, when it will snap into the shape of a closed circular hoop.

In spite of this apparent difference between the configuration of cables and rods at ($u = 2\pi$, $v = 0$), it should be borne in mind that this point does in any case represent the limit of stability of the straight form, for rods as well as for cables. This behavior of rods is also discussed in Ref. 2 (paragraph 272(d), page 417), where the more general Greenhill formula is also developed.

The main purpose in presenting Fig. 2 is to show the central importance of Greenhill's formula in evaluating the stability of rods and cables and to clarify its relationship

to the force-only solution of Southwell. The figure may also be used to determine the possible configuration (or configurations, in the double-valued region between the two branches of the Greenhill parabola) when end loads u and v to the left of the $\gamma = 0$ branch are given. In this application the barrier locus $D_{min}/L = 0$ must be taken into account.

Why Don't Rods or Solid Wires Hockle?

Understanding the fundamental importance of Greenhill's formula, we can now evaluate under what conditions a rod in combined tension and torsion might undergo bending. For a circular steel rod or wire with Young's modulus of $E = 2 \times 10^{11}\ N/m^2$ and shear yield strength $\sigma_s = 350 \times 10^6\ N/m^2$, the precondition for bending is that any tension be so low as to keep the actual tensile stress below $0.15 \times 10^6\ N/m^2$, or a mere 0.027% of a tensile capability of say $550 \times 10^6\ N/m^2$! Any attempt to induce bending due to twist at a higher applied tension will cause torsion shear failure instead. This conclusion is arrived at as follows: Assume the tension T is such that bending is just possible by Greenhill's formula while at the same time the shear yield limit $\sigma_s = 2M/\pi a^3$ is reached, where a is the radius of the rod. For an infinitely long rod (which would bend most readily), Greenhill's formula reduces to $M = 2(TEI)^{1/2} = a^2\ (TE\pi)^{1/2}$. In the equation $\sigma_s = 2M/\pi a^3$, if $a^2(TE\pi)^{1/2}$ is substituted for M and then σ_t (the tensile stress produced) is substituted for $T/\pi a^2$, the result yields

$$\sigma_t = \sigma_s^2/4E.$$

For the values assumed earlier this becomes $\sigma_t \doteq 0.15 \times 10^6\ N/m^2$ as stated.

EXPERIMENTAL VERIFICATION

Woods Hole, MIT, and NCEL Tests

Field and laboratory tests to investigate the hockling or "kinking" properties of oceanographic cable have been conducted by Berteaux and Walden [9], Vachon [10], and Liu [11,12]. The laboratory tests by Berteaux and Walden concentrated on measuring rotation rather than torque and therefore cannot be evaluated here without further knowledge of the torsional rigidity.

Vachon measured for a number of cables and loadings the bending stiffness EI (which he was also able to calculate to good accuracy), the force T, and the hockling torque M both positive (tightening) and negative (unwinding). His 17-foot cables were long enough to reduce end effects to a modest 10% or so (which can be estimated from Greenhill's formula, equation (11), by dividing through by ℓ^2 and comparing π^2/ℓ^2 to T/EI or $(M/2EI)^2$). This order of magnitude is borne out also by his evaluation of the effects of end mounting on hockling torque.

When these end effects are ignored, by putting $\ell = \infty$ in the Greenhill formula, and each experiment is assigned a Greenhill number $G = M^2/4TEI$, which theoretically should

then be equal to 1, the results show G to range from about 0.5 to 5. These values of G are at least the right order of magnitude and have a reasonable mean value. Vachon's results show a rather wide divergence in positive and negative hockling torques for identical cables and tensions. If the torque had been consistently higher in the positive direction, one might be tempted to ascribe this divergence to increased bending stiffness for a tightened cable. Unfortunately, in half the cases the divergence goes the other way, and no explanation can be offered at this time. In the one case in which the positive and negative hockling torques fell to within about 10% of their average, the calculated Greenhill numbers turn out to be an encouraging 1.2 and 0.8 respectively.

Reading about the explosiveness with which some of Vachon's kinking experiments were completed ("a 3/16 inch thick circular ring, supporting the water can [used as a weight], was straightened out, thus dropping the can to the floor") is more amusing than having observed it. But in retrospect, this explosive behavior is quite consistent with the instability of the Greenhill loading in tension combined with the substantial energy stored in the relatively stiff twisting mode at the instant when that instability occurs. Vachon was well aware of the energy consideration, which he reviewed in his report.

The applicability of Greenhill's formula to cable kinking has also been recognized by Liu, whose recent experiments described in Ref. 11 appear to be well correlated with the formula over a wide loading range. Reference 12 is a more detailed report of his results.

NRL Tests

In view of the incomplete knowledge at NRL about the conditions under which hockling tests at other Laboratories were conducted and, even more, to get some engineering insight into the phenomena under study, a few rudimentary measurements were made at NRL. The most sophisticated measuring tools were fish scales and weights (steel, to keep the floor dry), and the time allowed for the experiment was about 2 days. Nonetheless the results showed good agreement with theory.

The no-load small-deflection bending stiffness EI for a sample of 1/4-inch cable was measured by loading several lengths of it as cantilever beams. Calculated values of EI ranged from 0.051 to 0.063 Nm^2, with 0.057 as an average value.

A 12-foot length of this cable was suspended from the ceiling. Because of a lack of safety devices, loading was kept at light tensions up to 18 N and torques up to 2 Nm. For four loading cases Greenhill numbers $G = M^2/4TEI$ of 0.97, 0.72, 0.97, and 1.10 were obtained. In view the closeness of these observations to the theoretical $G = 1$, more comprehensive and better instrumented hockling experiments are anticipated, with a view to developing a usable method for specifying torsional properties of marine cables.

CONCLUSIONS

The conclusions are as follows:

- The importance of Greenhill's 100-year-old formula for determining the elastic stability of rods in combined axial force and twist far surpasses merely defining the onset

of possible bending modes. For rods or cables in tension the Greenhill condition represents the largest torque which can be applied for a given tension or the lowest tension capable of supporting a given torque. It represents a point of instability with respect to increasing torque or decreasing tension. This instability is expected to be particularly violent in the case of cables, because of their low bending stiffness.

- For applications in which a cable remains substantially straight, such as the lifting of objects from the ocean floor, the Greenhill formula, modified by an appropriate safety factor, should provide a valid criterion for estimating the onset of hockling.

- For applications involving initially curved cables, such as in towing, where the influences of gravity and drag are strong, the situation is less clear. It is known that the Greenhill condition cannot be exceeded, but it is possible that instabilities might already occur at much lower values of torque or higher values of tension, since the relevance of Greenhill's formula depends on the relatively large twisting stiffness of the initially unbent and straight cable. The stability of initially curved cables thus is an interesting subject for further research.

ACKNOWLEDGMENTS

Major credit is due to James Mark, who wrote the Fortran routine with great care, taking into account the several complications which occurred before it ran well for all cases of interest. The author is also grateful for numerous fruitful discussions held on both the practical and theoretical aspects of the cable hockling problem with George J. O'Hara and Richard A. Skop, both of the NRL Ocean Technology Division. The experimental checks performed on this problem at NRL were carried out most competently, in a short time and using only the simplest tools, by Darrell A. Milburn and Kenneth M. Ferer.

This work has been completed as part of the Division's broader research program on the use of cables in ocean engineering, and has been supported in part by the Office of Naval Research program in Structural Mechanics.

REFERENCES

1. G. Kirchhoff, Jour. of Math. (Crelle) 56 (1859).

2. A.E.H. Love, *The Mathematical Theory of Elasticity*, Dover, New York, 1944, Chapters XVIII and XIX.

3. A.G. Greenhill, Proc. Inst. Mech. Engrs. (London), 1883.

4. S.P. Timoshenko and J.M. Gere, *Theory of Elastic Stability*, 2nd edition, McGraw-Hill, Toronto and London, 1961, p. 157.

5. R.V. Southwell, *An Introduction to the Theory of Elasticity*, Dover, New York, 1969, Chapter XIII.

6. M.F. Massoud, and Y.A. Youssef, "Dynamic Characteristics of Thin Space Beams," The Shock and Vibration Digest 5 (No. 1), 7-14 (Jan. 1973).

7. A.E. Green, P.M. Naghdi, and M.L. Wenner, "On the Theory of Rods, Part I: Derivations from the Three-Dimensional Equations," and "Part II: Developments by Direct Approach," Reports AM-73-4/5 to the Office of Naval Research on Project NR 064-436, University of California, Berkeley, Oct./Nov. 1973.

8. L.O. Landau, and E.M. Lifshitz, *Theory of Elasticity*, Pergammon Press, London/Paris/Frankfurt, 1959, Sections 17-19.

9. H.O. Berteaux, and R.G. Walden, "Analysis and Experimental Evaluation of Single Point Moored Buoy Systems," unpublished manuscript 69-36, Woods Hole Oceanographic Institution, under Office of Naval Research Contract N00014-66-C0241, NR 083-004, May 1969.

10. W.A. Vachon, "Kink Formation Properties and Other Mechanical Characteristics of Oceanographic Strands and Wire Rope," Technical Report E-2497 prepared at the MIT Charles S. Draper Laboratory for Woods Hole Oceanographic Institution under Office of Naval Research Contract N00014-66-C0241, NR 083-004, Apr. 1970.

11. F.C. Liu, "Rotational and Kinking Properties of *EM* Cables," abstracts of papers presented at "Civil Engineering in the Ocean/III," American Society of Civil Engineers, New York, June 1975.

12. F.C. Liu, "Kink Formation and Rotational Response of Single and Multistrand Electromechanical Cables," CEL Technical Note L-1403, Naval Civil Engineering Laboratory, Port Hueneme, Calif., Oct. 1975.

Appendix A
FORTRAN LISTINGS FOR PROGRAM HOCKLE

```
      PROGRAM HOCKLE
      DIMENSION FF(100),GAM(100),DSD(100)
      COMMON /BLOK1/ A11,A12,A13,F,H,PHI,A,B,C,X,Y,Z,S,A21,A31,A32,A33,
     1 DA,DB,DC,DX,DY,DZ,ABAR,ICOUNT,HNEW,A110,GAMMA,IBFLAG
      COMMON /BLOK2/ KOUNT,SYMPHI(1500),SYMALF(1500)
      COMMON /BLOK3/ XS(1500),YS(1500),ZS(1500),LS,DMIN,DS,SMIN,NSTEPS
      REAL L
C THE FOLLOWING  FUNCTION CALCULATES GREENHILL LENGTH
C
      XL(X)= 6.2831853072 / SQRT(X * (X-4))
C
C  WE NOW READ IN NUMBER OF STEPS BETWEEN PRINT LINES AND STOP SIGNAL
C
      DS=0.
 1001 READ 2,NSTEPS,NSTOP
    2 FORMAT(2I5)
      IF(NSTOP.GT.0) GO TO 1002
C
C WE NOW READ IN UP TO 100 POINTS F,GAMMA,DS
C
      READ 1,NF,NG,NDS,(FF(I),I=1,NF),(GAM(J),J=1,NG),(DSD(KQ),KQ=1,NDS)
    1 FORMAT(3I3,(/8F10.0))
C
C FOR OUTPUT PURPOSES WE NOW COMPUTE THE LENGTH OF THE PRINT INTERVAL
C AND INTIALIZE THE CASE NUMBER.  PRN,NCASE
C
      NCASE=0
C
C WE RUN THE CALCULATION THROUGH THE F,GAMMA PAIRS.
C        I IS A COUNTER TO STEP THROUGH  THE VALUES OF F
C        J IS A COUNTER TO STEP THROUGH THE VALUES OF GAMMA
C KQ IS A COUNTER TO STEP THE VALUES OF DS
C
      DO 1000 I=1,NF
      F=FF(I)
C
C *** CHECK TO SEE IF F IS IN THE INTERVAL (0,4) AND IGNORE GREENHILL
C     LENGTH IF IT IS.
C
      GL=0
      IF( (F.LT.0.) .OR. (F.GT.4.) ) GL=XL(F)
C
C *** INNER LOOPS ON J AND KQ
C
      DO 1000 J=1,NG
      DO 1000 KQ=1,NDS
      DS=DSD(KQ)
      PRN=NSTEPS*DS
      IBFLAG=0
      SM=0.
      SMIN=0.
      R=0.
      RN=0.
      NPRINT=0
      NTEST=0
      S1=S2=S3=S4=0.
```

```
      ABAR=0
      KOUNT=1
      MIDFLAG=0
      ICOUNT=0
      GAMMA=GAM(J)
C
C *** PRINT INITIAL CONDITIONS AND CASE NUMBER
C
      NCASE=NCASE+1
      PRINT 3,NCASE
    3 FORMAT(1H1,*-----------INITIAL CONDITIONS FOR CASE NO.*,I4//)
      PRINT 4,F,GAMMA
    4 FORMAT(1H ,10X,*F=*,F6.2,10X*GAMMA=*,F6.2//)
C
C *** CONVERT GAMMA TO RADIANS FOR CALCULATION
C
      GAMMA=GAMMA * 3.1415926536/180.
C
C INITIALIZE ANGLES AND COORDINATES
C
      A=1.5707963268
      B=0.
      C=GAMMA
C
C     ALTERNATE INITIAL VALUES A=0., B=GAMMA, C=0. INSERT HERE
C
      X=0.
      Y=0.
      Z=0.
      S=0.
      RNMAX=0.
      RN=0.
      LS=1
C
C CALCULATE INITIAL VALUES OF A(I,J)
C
      A11=COS(B)*COS(C) $ A110=A11
      ABAR=ABAR+A11
      ICOUNT=ICOUNT+1
      A12=COS(B)*SIN(C)
      A13=-SIN(B)
      A21=SIN(A)*SIN(B)*COS(C) - COS(A)*SIN(C)
      A31=COS(A)*SIN(B)*COS(C) + SIN(A)*SIN(C)
      A32=COS(A)*SIN(B)*SIN(C) - SIN(A)*COS(C)
      A33=COS(A)*COS(B)
C
C ALTERNATE A21 INSERT HERE
C
C CALCULATE INITIAL VALUES OF H,PHI,PSI,HNEW
C
      H=A31-Z*A32+Y*A33
      PHI=0.5*A11
      IF(ABS(H)-.00000001)  5,5,41
   41 PHI= A11 - A31 / (F*H)
    5 HNEW=SQRT((2./F)*(COS(GAMMA)-A11) + SIN(GAMMA)**2)
      PSI=0.
```

```
      ITEST1=0
      ITEST2=0
      PUNCH 60,NCASE
   60 FORMAT(* INITIAL VALUES FOR CASE*,I6)
      PUNCH 61,S,PHI,X,Y,Z,RN,H,PSI
      PUNCH 61,A11,A12,A13,A21,A31,A32,A33
   61 FORMAT(8F10.6)
   64 FORMAT(4F14.8,2F12.6)
C
C PRINT OUTPUT HEADERS.
C
      PRINT 6
    6 FORMAT(/4H    S,6X,3HPHI,5X,1HX,6X,1HY,6X,1HZ,5X,2HH0,5X,3HPSI,5X,
     11HA,6X,1HB,6X,1HC,6X,1HH,5X,3HA11,4X,3HA12,4X,3HA13,4X,3HA21,4X,
     23HA31,4X,3HA32,4X,3HA33/)
      IFFL=1
C
C PRINT INITIAL VALUES
C
    7 PRINT 8,S,PHI,X,Y,Z,HNEW,PSI,A,B,C,H,A11,A12,A13,A21,A31,A32,A33
    8 FORMAT(1H ,18F7.4)
      RN=SQRT(Y**2+Z**2)
      IF(RN.GT.RNMAX) RNMAX=RN
      NPRINT=NPRINT+1
      CALL SYM(IFFL,S1,S2,S3,S4)
      IF(MIDFLAG.EQ.1) GO TO 50
      XS(LS)=X
      YS(LS)=Y
      ZS(LS)=Z
      LS=LS+1
   50 CONTINUE
C
C WE NOW BEGIN TO STEP OUR WAY ALONG THE CABLE
C          ISP IS A COUNTER CONTROLLING PRINT
C WE ARE LOOKING FOR MIDHOCKLE
C
      ISP=0
    9 A21OLD=A21
      CALL STEP(DS)
C
C *** TEST FOR MIDHOCKLE
C
      IF(((A21*A21OLD).LT.0).AND.(MIDFLAG.EQ.0))GO TO 13
C
C *** WE ARE NOT AT MIDHOCKLE.
C     HAVE WE COME TO THE END OF THE CABLE.
C
   10 IF(MIDFLAG.NE.1) GO TO 11
      IF(NPRINT.EQ.NTEST)  GO TO 25
C
C WE ARE NOT AT END OR MIDHOCKLE. CHECK TO SEE IF IT IS TIME TO PRINT
C
   11 ISP=ISP+1
C
C WE ARE NOW READY TO PRINT SO WE NEED TO COMPUTE HNEW,PSI,PHI
      PSI=ASIN( Z/SQRT(Z**2+Y**2) )
```

```
      IF(ISP.LT.NSTEPS)  GO TO 9
      GO TO 7
C
C WE ARE AT MIDHOCKLE
C
   13 PRINT 14
   14 FORMAT(////1H ,*---------- MIDHOCKLE ----------*///)
C
C WE NOW FIND SM THE LENGTH TO MIDHOCKLE
      MIDFLAG=1
      DSM=-A21*DS/(A21-A21OLD)
      CALL STEP(DSM)
      RN=SQRT(Y **2+Z **2)
      IF(RN.GT.RNMAX) RNMAX=RN
      XS(LS)=X
      YS(LS)=Y
      ZS(LS)=Z
C
C CURRENT VALUES OF VARIABLES IN COMMON ARE MIDHOCKLE VALUES SM=S.
      PSI=ASIN( Z/SQRT(Z**2 +Y**2) )
      L=2.*S
      FL=F*L
      FL2=FL*L
C
C CALCULATE TWIST AND BENDING ENERGIES
C
      ENTL=F*FL2*0.5*COS(GAMMA)**2
      ABAR=ABAR/ICOUNT
      ENB=FL2*COS(GAMMA)+F*FL2*0.5*SIN(GAMMA)**2-FL2*ABAR
      SM=S
      G=GAMMA*180./3.1415926536
C
C PRINT HEADERS FOR MIDHOCKLE PRINTOUT
C
      PRINT 17
   17 FORMAT(1H ,32X *PRINT*,3X,*GREENHILL* ,2X,*MIDHOCKLE*,3X,*TOTAL*,
     124X,*TWIST  BENDING*)
      PRINT 18
   18 FORMAT(1H ,5X,*F*,6X,*GAMMA*,4X,*STEP SIZE INTERVAL *,3(*  LENGTH
     1 *),4X,*FL*,6X,*FL2*,7X,*ENERGY   ENERGY*)
      PRINT 19,F,G    ,DS,PRN,GL,SM,L,FL,FL2,ENTL,ENB
   19 FORMAT(1H ,F10.5,F7.2,3X,9F10.5////)
C
      CALL DCALC
      PRINT 51 DMIN,SMIN
   51 FORMAT(1H ,*  MINIMUM DISTANCE IS  *,F12.6,*     S=*,F12.5)
      PRINT 6
      PRINT 8,S,PHI,X,Y,Z,HNEW,PSI,A,B,C,H,A11,A12,A13,A21,A31,A32,A33
      PUNCH 62
   62 FORMAT(*MIDHOCKLE*)
      PUNCH 61,F,GAMMA,DS,PRN,GL,L,RN,RNMAX
      PUNCH 64,FL,FL2,ENTL,ENB,DMIN,SMIN
      PUNCH 61,S,PHI,X,Y,Z,RN,H,PSI
      PUNCH 61,A11,A12,A13,A21,A31,A32,A33
C
C WE NOW GO TO POINT SYMMETRIC WITH PRINT VALUE PRECEEDING MIDHOCKLE
```

```
C
      DS2=DS+DSM
      CALL STEP(DS2)
      K2=ISP
      DO 24 K=1,K2
   24 CALL STEP(DS)
      CALL SYM(IFFL,S1,S2,S3,S4)
      PRINT 8,S,PHI,X,Y,Z,HNEW,PSI,A,B,C,H,A11,A12,A13,A21,A31,A32,A33
C
C CALCULATE TO END OF CABLE WITH ORIGINAL DS
C
      IFFL=2
      ISP=0
C
C SAVE NUMBER OF STEPS TO MID HOCKLE FOR END TEST
C
      NTEST=NPRINT
      NPRINT=1
      GO TO 9
C
C WE GET TO HERE WHEN WE HAVE COME TO THE END OF THE CABLE
C
   25 IFFL=3
      CALL SYM(IFFL,S1,S2,S3,S4)
      PRINT 26,S1,S2
   26 FORMAT(////,1H ,*---SYMMETRY FACTORS---*,6X,*PHI*,F12.5,10X,*A21*,
     1F12.5///)
      PRINT 27,S3,S4
   27 FORMAT(//1H ,*---RMS VALUES---*,15X,F12.5,13X,F12.5)
      R1=S1/S3
      R2=S2/S4
      PRINT 28,R1,R2
   28 FORMAT(//1H ,*----RATIOS----*,18X,F12.5,13X,F12.5)
      PUNCH 63
   63 FORMAT(*ENDHOCKLE*)
      PUNCH 61,S,PHI,X,Y,Z,RN,H,PSI
      PUNCH 61,A11,A12,A13,A21,A31,A32,A33
      PUNCH 61,S1,S2,S3,S4,R1,R2
C
C IF THERE ARE MORE PAIRS F,GAMMA TO BE DONE RESET AND GO AGAIN
C OTHERWISE STOP.
C
 1000 CONTINUE
      GO TO 1001
 1002 STOP
      END
```

```
      SUBROUTINE DCALC
      COMMON /BLOK3/ XS(1500),YS(1500),ZS(1500),LS,DMIN,DS,SMIN,NSTEPS
      DIMENSION D(1500)
C
C     THIS SUBROUTINE CALCULATES RELATIVE MINIMUM OF DISTANCE
C     BETWEEN PAIRS OF SYMMETRIC POINTS
C
      DO 5 J=1,1500
    5 D(J)=-1.
      DMIN=-2.
      XM=XS(LS)
      YM=YS(LS)
      ZM=ZS(LS)
      MIN=LS-1
      MM=-1
      DO 10 I=1,MIN
      J=LS-I
      T =YM**2+ZM**2
      IF(T1.LT.0.0000001) GO TO 10
      T2=(YM*ZS(J)-ZM*YS(J))**2
      T3=(XM-XS(J))**2
      D(J)=2.*SQRT(T2/T1+T3)
   10 CONTINUE
      PRINT 100, (D(J), J=1,LS)
  100 FORMAT(1H ,10F10.5)
C
C LOOK  FOR MINIMA
C
      TEST=D(LS)
      I=LS-1
   20 IF(D(I).LT.TEST) GO TO 30
      TEST=D(I)
   21 I=I-1
      IF(I.EQ.0) GO TO 50
      GO TO 20
   30 IF(D(I).LT.0.) GO TO 21
   31 TEST=D(I)
      I=I-1
   32 IF(D(I).GT.TEST) GO TO 40
      TEST=D(I)
      I=I-1
      GO TO 32
   40 DMIN=TEST
      SMIN=( I+1)*DS*NSTEPS
      RETURN
   50 PRINT 51
   51 FORMAT(1H,*    D HAS NO RELATIVE MINIMUM    *)
      RETURN
      END
```

```
      SUBROUTINE SYM(I,S1,S2,S3,S4)
      COMMON /BLOK1/ A11,A12,A13,F,H,PHI,A,B,C,X,Y,Z,S,A21,A31,A32,A33,
     1 DA,DB,DC,DX,DY,DZ,ABAR,ICOUNT,HNEW,A110,GAMMA,IBFLAG
      COMMON /BLOK2/ KOUNT,SYMPHI(1500),SYMALF(1500)
C
C THIS SUBROUTINE IS CALLED AT EACH PRINT STEP AND CALCULATES SYMMETRY
C FACTORS AT END OF CABLE.
C
      GO TO (1,2,3),I
C
C WE ARE STILL IN FIRST HALF OF CABLE, WE ACCUMULATE VALUES OF PHI,A21
C
    1 SYMPHI(KOUNT)=PHI
      SYMALF(KOUNT)=A21
      S3=S3+PHI**2
      S4=S4+A21**2
      KOUNT=KOUNT+1
      J=KOUNT
      IF(KOUNT.EQ.1500) GO TO 5
      RETURN
C
C WE ARE IN SECOND HALF OF CABLE.
C
    2 KOUNT=KOUNT-1
      SYMPHI(KOUNT)=(SYMPHI(KOUNT)-PHI)**2
      SYMALF(KOUNT)=(SYMALF(KOUNT)+A21)**2
      S1=S1+SYMPHI(KOUNT)
      S2=S2+SYMALF(KOUNT)
      RETURN
C
C WE ARE AT THE END OF THE CABLE.
C
    3 S1=SQRT(S1 /(J+1) )
      S2=SQRT(S2/(J+1))
      S3=SQRT(S3/(J+1))
      S4=SQRT(S4/(J+1))
      RETURN
    5 PRINT 4
    4 FORMAT(1H ,*----------KOUNT OVER LIMIT----------*)
      STOP
      END
```

```
      SUBROUTINE STEP(DS)
      COMMON /BLOK1/ A11,A12,A13,F,H,PHI,A,B,C,X,Y,Z,S,A21,A31,A32,A33,
     1 DA,DB,DC,DX,DY,DZ,ABAR,ICOUNT,HNEW,A110,GAMMA,IBFLAG
      COMMON /BLOK2/ KOUNT,SYMPHI(1500),SYMALF(1500)
C THIS SUBROUTINE STEPS THE CALCULATION FROM POINT TO POINT ALONG THE
C     CABLE.
C             INPUT  DS = STEP SIZE FOR CALCULATION
C             -----
C
C             OUTPUT   A  -
C             ------   B   -ANGLES
C                      C  -
C                      X  +
C                      Y   +COORDINATES
C                      Z  +
C
C CHECK IBFLAG.
C                IBFLAG=1 IF B TOO CLOSE TO 90 DEGREES
C IF B TOO CLOSE TO 90 WE USE ALTERNATE FORMULAE
C
      IF(IBFLAG.EQ.1) GO TO 500
C
C*** FIRST WE CALCULATE INCREMENTS FOR ANGLES
C
  400 DB= (-F * H * SIN(A)) * DS
      DA= F * (H * COS(A) * SIN(B)/COS(B) + PHI) * DS
      DC= ( F * H * COS(A)/COS(B)) *DS
C
C *** WE ADD INCREMENTS TO PRESENT VALUES TO OBTAIN VALUES
C     AT A POINT DS-UNITS FURTHER ALONG THE CABLE.
C
      A=A+DA
      B=B+DB
      C=C+DC
      SIGNCB=SIGN(1.,COS(B))
C
C CHECK TO SEE IF B TOO CLOSE TO 90 DEGREES
C
      CB=ABS(COS(B))
      IF(CB.LT.0.1     ) GO TO 100
C
C ** WE ARE NOW READY TO CALCULATE INCREMENTS TO COORDINATES
C
      DX=A11 * DS
      DY=A12 * DS
      DZ=A13 * DS
C
C *** WE ADD THE INCREMENTS TO THE PREVIOUS VALUES OF THE COORDINATES
C     TO OBTAIN VALUES AT A POINT DS-UNITS FURTHER ALONG THE CABLE.
C
      X=X+DX
      Y=Y+DY
      Z=Z+DZ
C
C *** FINALLY,WE CALCULATE NEW CABLE LENGTH
C
```

```
      S=S+DS
C
C *** CALCULATE NEW VALUES FOR A21,A31,A32,A33
C
      A21=SIN(A)*SIN(B)*COS(C) - COS(A)*SIN(C)
      A31=COS(A)*SIN(B)*COS(C) + SIN(A)*SIN(C)
      A32=COS(A)*SIN(B)*SIN(C) - SIN(A)*COS(C)
      A33=COS(A)*COS(B)
C
C *** CALCULATE NEW VALUES FOR A11,A12,A13 AND RETURN
C
      A11=COS(B)*COS(C)
      A12=COS(B)*SIN(C)
      A13=-SIN(B)
  300 H= A31-Z*A32+Y*A33
      ABAR=ABAR+A11  $  ICOUNT=ICOUNT+1
      PHI=0.5*A110
      IF(ABS(H)-.00000001) 16,16,15
   15 PHI=A110-A31/(F*H)
   16 HNEW=0.
      HNEW2=(2./F)*(COS(GAMMA)-A11)+SIN(GAMMA)**2
      IF(HNEW2.GT.0.)HNEW=SQRT(HNEW2)
      RETURN
C
C GET TO HERE WHEN B TOO CLOSE FOR FIRST TIME.
C  WE SAVE CURRENT VALUES OF A,B,C
C  WE THEN SET IBFLAG=1, A=C=0.
C
  100 ASAVE=A
      BSAVE=B
      CSAVE=C
      IBFLAG=1
      A=0.
      C=0.
      B=0.
C
C NOW CALCULATE A22,A23 FOR FIRST TIME THROUGH
C
      A22=SIN(ASAVE)*SIN(BSAVE)*SIN(CSAVE)+COS(ASAVE)*COS(CSAVE)
      A23=SIN(ASAVE)*COS(BSAVE)
C
C WE NOW CALCULATE NEW VALUES FOR X,Y,Z,S
C
  200 X=X+A11*DS
      Y=Y+A12*DS
      Z=Z+A13*DS
      S=S+DS
C
C NOW CALCULATE NEW VALUES OF A(I,J)
C
      DA11=F*H*A21*DS
      DA12=F*H*A22*DS
      DA21=F*(-H*A11+PHI*A31)*DS
      DA13S=F*H*A23*DS
      DA31S=-F*PHI*A21*DS
      DA32S=-F*PHI*A22*DS
```

```
      A11=A11+DA11
      A12=A12+DA12
      A21=A21+DA21
      A13S=A13+DA13S
      A13=SQRT(1.-A11**2-A12**2)
      IF((A13*A13S).LT.0.) A13=-A13
      A31S=A31+DA31S
      A31=SQRT(1.-A11**2-A21**2)
      IF((A31*A31S).LT.0.) A31=-A31
      A23=(-A11*A13*A21+A12*A31)/(1.-A11**2)
      A22=(A12*A23-A31)/A13
      A32S=A32+DA32S
      A32=SQRT(1.-A12**2-A22**2)
      IF((A32*A32S).LT.0.) A32=-A32
      A33=(A11*A31+A12*A32)/(-A13)
      GO TO 300
C----------
C
C GET TO HERE IF IBFLAG=1
C CHECK TO SEE IF B STILL TOO CLOSE.
C
  500 CB=SQRT(1.-A13**2)
      IF(CB.LT.0.1    ) GO TO 200
C
C GET TO HERE IF B NO LONGER TOO CLOSE.
C WE RESTORE B TO PROPER QUADRANT
C
      IF(SIGNCB.GT.0.) CB=-CB
      SB=-A13
C
C DETERMINE B,A,C UP TO ROTATIONS
C
      B=ACOS(CB)
      IF(SB.LT 0.) B=-B
      CA= A33/CB
      SA=A23/CB
      A=ACOS(CA)
      IF(SA.LT.0.) A=-A
      CC=A11/CB
      SC=A12/CB
      C=ACOS(CC)
      IF(SC.LT.0.) C=-C
C
C NOW DETERMINE ROTATIONS
C
      BROT=(B-BSAVE)/6.283185307
      N=BROT+0.5
      B=B+N*6.283185307
      CROT=(C-CSAVE)/6.283185307
      N=CROT+0.5
      C=C+N*6.283185307
      AROT=(A-ASAVE)/6.283185307
      N=AROT+0.5
      A=A+N*6.283185307
C
C CONTINUE WITH ORIGINAL FORMULAE
C
      IBFLAG=0
      GO TO 400
      END
```